JOBWORKS

ADULT LEARNING CENTRE

INFORMATION AND PREPARATION FOR PRE-CALCULUS MATHEMATICS (40S)

For the best chance of success in this course, please be prepared to attend classes, study, and do homework regularly.

Students taking this course are **required** to have a scientific calculator. Staples, Walmart, and Amazon.ca have these calculators for sale.

Only calculators can be used during quizzes, tests, and exams. Staff will not have any spare calculators to loan.

The use of cell phones/electronic devices as a calculator will not be permitted.

Teachers will focus on covering the content in the curriculum of this course. Limited time will be spent on reviewing skills and concepts that should have already been acquired by this level. To prepare for this course in advance, being familiar with the following topics would be beneficial:

- Operations with Signed Numbers
- Fractions
- Order of Operations
- Basic Exponents
- Basic Algebra
- Trigonometry
- Operations with Radicals
- Factoring and Distributing

A basic review on most of the topics listed above is attached. Visit www.khanacademy.org for tutorials. Additional resources can also be found online.

Algebra 1 for Dummies, 2nd Edition, by Mary Jane Sterling (ISBN 978-0-470-55964-2) is a book that covers many of the above topics. It can be ordered online from Amazon.ca or Chapters.

Free tutoring may be available if necessary. Visit the front desk staff to sign up after attending at least one class. Note that there is always a high demand for the limited number of spots. Students will be accommodated on a first come, first served basis if a tutor is available. Students interested in paying for a tutor can contact the Association of Independent Tutors at 204-226-3437 or at www.independenttutors.com.

Contact us at (204) 453-8351 or visit our website at www.jobworksschool.com if there are any questions or concerns.

Operations with Signed Numbers

Numbers can be positive or negative

A. Adding and Subtracting

Examples:

3 – 5 = -2
-4 + 10 = 6
-2 – 5 = -7   (can be thought of as -2 + -5 = -7)

B. Multiplying and Dividing

Rules:

- positive number multiplied or divided by a positive number equals positive answer.
- positive number multiplied or divided by a negative number (or vice versa) equals a negative answer.
- negative number multiplied or divided by a negative number equals a positive answer.

Examples:

-2 x 3 = -6
-5 ÷ 1 = -5
-10 x -2 = 20

Fractions

$$\frac{3}{4} \begin{matrix} \rightarrow \text{numerator} \\ \rightarrow \text{denominator} \end{matrix}$$

To add or subtract fractions, the denominator must be the same, or be equated to the same.

Example:

$$\frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$$

Example:

$$\frac{1}{3} + \frac{1}{2} = ?$$

- multiply 3 x 2 = 6

$$\frac{1}{3} + \frac{1}{2} = \frac{?}{6}$$

- cross multiply 2 x 1, and 3 x 1, and put the + sign between these two numbers.

$$\frac{1}{3} + \frac{1}{2} = \frac{2+3}{6} = \frac{5}{6}$$

Example:

$$\frac{5}{6} - \frac{1}{2} = \frac{10-6}{12} = \frac{4}{12} = \frac{1}{3}$$

* Always put your answer in lowest terms

To multiply fractions, multiply the numbers on the top, and then the two numbers on the bottom.

$$\frac{2}{3} \times \frac{1}{2} = \frac{2 \times 1}{3 \times 2} = \frac{2}{6} = \frac{1}{3}$$

To divide fractions, the second fraction must be inverted, and then the two fractions are multiplied.

$$\frac{4}{6} \div \frac{1}{3} = \frac{4}{6} \times \frac{3}{1} = \frac{12}{6} = 2$$

The inverted fraction is known as a "reciprocal".

Order of Operations

When math questions involve combinations of addition, subtraction, multiplication and division, a certain order must be followed to obtain the correct answer. We use the acronym BEDMAS.

Brackets The order is from brackets down to subtraction
Exponents
Division
Multiplication
Addition
Subtraction

Example:

$2 - 3 + 4 \div 2 + 3 \times 4 - 5$

$2 - 3 + \underbrace{4 \div 2} + \underbrace{3 \times 4} - 5$

$\underbrace{2 - 3} + \underbrace{2 + 12} - 5$

$-1 + 14 - 5$

$= 8$

Example:

$\underbrace{(2 - 4)}^2 + 4 + 1 - \underbrace{8 \div 2}$

$(-2)^2 + 4 + 1 - 4$

$4 + 5 - 4$

$= 5$

Basic Exponents

The makeup of exponential numbers is as follows:

$$whole\ number \rightarrow 3\ \underbrace{x}_{base}\ ^{2 \leftarrow exponent}$$

Sometimes such an expression is written as $3(x)^2$, but means the same thing.

Example:
$3^2 = 3 \times 3 = 9$

Basic Rule of Exponents

1. Addition/Subtraction Rule – exponential expressions that have the same base AND the same exponent can be added or subtracted. It is only the whole number that is then considered.

   Example: $2x^3 + 4x^3 = 6x^3$

   Example: $4x^5 - 3x^2 + 2x^5 + 10x^2 = 6x^5 + 7x^2$

2. Multiplication Rule – when multiplying exponential expressions, the common base idea is still required. However, this time the exponents are simply added together. The whole numbers are still multiplied as per normal.

   Example: $(3x^2)(2x^5) = 6x^7$

   Example: $(-2x^2)(3y^3)(4x^3)(2y^2) = -48x^5y^5$

3. Division Rule – again, the common base idea is used. To divide exponential expressions, the exponents are subtracted. The whole numbers follow normal division rules.

   Example:
   $\frac{8x^5y^3}{2x^2y^2} = 4x^3y^1\ or\ 4x^3y$

Practice Questions

Simplify each expression

1. $3x^2 - 2x^3 - 5x^2 + 3x^3$
2. $\frac{10x^5}{2x^3}$
3. $(3x^3)(2x^5)$
4. $3x^5 \div x^4$
5. $(4y^2)(2x^4)(-2y^3)(3x^3)$
6. $-8a^5 \div 2a^2$

Solutions

1. $-2x^2 + x^3$
2. $5x^2$
3. $6x^8$
4. $3x$
5. $-48x^7y^5$
6. $-4a^5$

Basic Algebra

Algebra involves the combination of both numbers and letters of the alphabet into mathematical equations, or sentences. The letter most often takes the place of some hidden number. The goal is to solve the equation and determine what this number is.

Example 1: Find what number "c" represents.
$2 + c = 5$

We read the above equation as "2 plus what equals 5".
The answer is obviously 3.

Example 2: Find what number "x" represents.
$3x = 6$

We read the above equation as "3 multiplied by what equals 6".
The answer is obviously 2.

The above examples are easy enough to solve in our head, and we do not have to solve them "algebraically". Solving algebraically involves isolating the letter on the left side of the equation and putting all numbers on the right side of the equation. This is also easy to do, but there are rules to follow.

Rule 1: "Rule of Opposites"
- the opposite of positive is negative (and vice versa)
- the opposite of multiplication is division (and vice versa)

Rule 2: What is done to one side of the equation must be done to the other.

Let's resolve the above two equations algebraically.

Example 1 solution:

$2 + c = 5$ - to isolate the "c" you must get rid of the positive 2 by subtracting 2.

$2 - 2 + c = 5 - 2$ - do it to both sides

$c = 3$ - the 2's on the left side cancel out, and we are left with the answer.

Example 2 solution:

$3x = 6$ - to isolate the "x" you must get rid of the 3 that the "x" is being multiplied by. (3x means 3 times x)

$\frac{3x}{3} = \frac{6}{3}$ - to do this, we must divide by 3 on both sides.

$\frac{\cancel{3}x}{\cancel{3}} = \frac{6}{3}$ - the 3's cancel out

$x = 2$

Example: Solve for "y"

$\frac{y}{3} = 4, \quad \frac{y}{3} \times 3 = 4 \times 3, \quad \frac{y}{\cancel{3}} \times \cancel{3} = 4 \times 3, \quad y = 12$

Example: Solve for "b"

$-4 + b = -7,$ $\quad -4 + 4 + b = -7 + 4,$ $\quad b = -3$

* Another name for the letters used in algebra is variables.

Practice Questions

A. Solve each variable

1. $3x - 4 = 5$
2. $2y + 4 = 8$
3. $7a + 2 = -12$
4. $-10c = 5$
5. $\frac{b}{10} - 2 = 8$
6. $-3 + 3m = -9$

B. Challenge Questions. Solve each variable.

1. $2x + 3x - 4 = 6$
2. $-2y + 5 = -4y + 7$
3. $\frac{-3a}{2} = -9$
4. $(3 - 5)^2 - 8d = -12$

Solutions

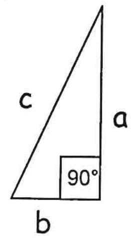
A. 1. $x = 3$ 2. $y = 2$ 3. $a = -2$ 4. $c = -\frac{1}{2}$ 5. $b = 100$ 6. $m = -2$

B. 1. $x = 2$ 2. $y = 1$ 3. $a = 6$ 4. $d = 1$

Trigonometry

Solving right triangles

A right triangle is one that has a 90° angle in it.
Example:



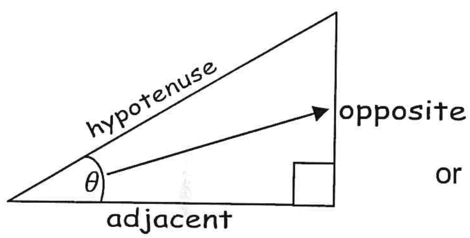
The three principle ratios that relate the angles with the lengths of the sides of the triangle are sine, cosine, and tangent. (sin, cos, tan)

$\sin\theta = \frac{opposite}{hypotenuse}$ $\cos\theta = \frac{adjacent}{hypotenuse}$ $\tan\theta = \frac{opposite}{adjacent}$

The hypotenuse will always be the longest side of the right triangle, which is side "c".

The side designated as adjacent (meaning beside) or the side designated as opposite depends on which angle in the right triangle that is being referred to. It is useful to draw an arrow from this angle of reference. (called theta, θ)

Example:



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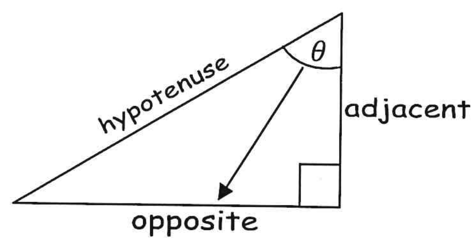
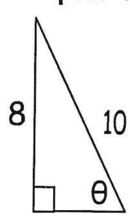


Example: Find the value of θ.



We will use sin because we know the opposite and hypotenuse.

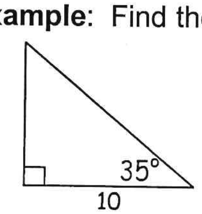
$\sin = \frac{opp}{hyp}$, $\sin = \frac{8}{10}$, $\sin = 0.8$

* Now press 2nd, sin on your calculator

$\theta = 53°$

What if we know θ, we know one side, and we want to find a second side?

Example: Find the hypotenuse of this right triangle.



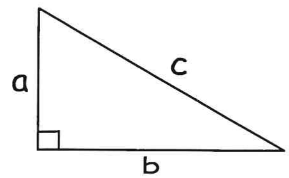
We have adjacent, and need the hypotenuse.
Use cos, and θ = 35°

$\cos\theta = \frac{adj}{hyp}, \cos 35° = \frac{10}{hyp}$

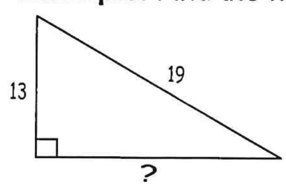
Press 35° , cos , = 0.819 $\quad 0.819 = \frac{10}{hyp}$, $hyp = \frac{10}{0.819}$, $hyp = 12.2$

If the 2 sides of a right triangle are known then the third side can always be found using the Pythagorean Theorem:

$$a^2 + b^2 = c^2$$



Example: Find the missing side.

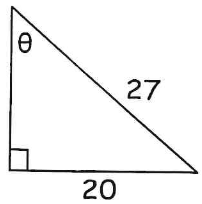


$$a^2 + b^2 = c^2$$
$$13^2 + b^2 = 19^2$$
$$169 + b^2 = 361$$
$$b^2 = 361 - 169$$
$$b^2 = 192$$
$$\sqrt{b^2} = \sqrt{192}$$
$$b = 13.9$$

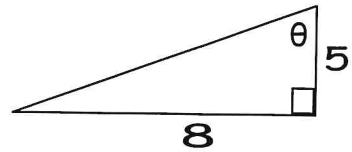
Practice Questions

A. Find the value of θ, and the missing side.

1.

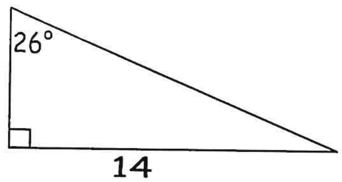


2.



B. Find a second side, then find the third side.

1.



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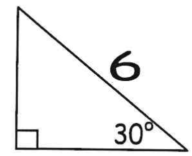


Solutions

A.

1. $\sin\theta = \frac{opp}{hyp}$, $\sin\theta = \frac{20}{27}$, $\boxed{\theta = 47.8°}$

$a^2 + b^2 = c^2$, $a^2 + 20^2 = 27^2$, $a^2 = 729 - 400$, $a^2 = 329$, $\boxed{a = 18.1}$

2. $\tan\theta = \frac{opp}{adj}$, $\tan\theta = \frac{8}{5}$, $\boxed{\theta = 58°}$

$a^2 + b^2 = c^2$, $5^2 + 8^2 = c^2$, $25 + 64 = c^2$, $89 = c^2$, $\boxed{c = 9.4}$

B.

1. $\sin 26° = \frac{14}{hyp}$, $0.4 = \frac{14}{hyp}$, $hyp = \frac{14}{0.4}$, $\boxed{hyp \text{ or "c"} = 35}$

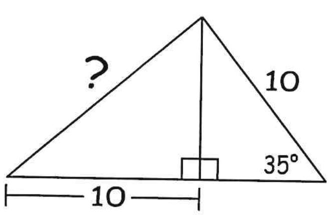
$a^2 + b^2 = c^2$, $a^2 + 14^2 = 35^2$, $a^2 = 1225 - 196$, $\boxed{a = 32.1}$

2. $\sin 30° = \frac{opp}{6}$, $0.5 = \frac{opp}{6}$, $\boxed{opp \text{ or "a"} = 3}$

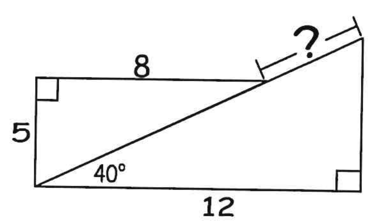
$a^2 + b^2 = c^2$, $3^2 + b^2 = 6^2$, $b^2 = 36 - 9$, $b^2 = 27$, $\boxed{b = 5.2}$

C. Challenge questions. Solve the ?

1.



2.



There are also three reciprocal trigonometric ratios. These are cosecant, secant, and cotangent (csc, sec, cot). Here is how they are related to the first three:

$$csc = \frac{1}{sin} \qquad sec = \frac{1}{cos} \qquad cot = \frac{1}{tan}$$

You will receive practice with these in the 40S Precalculus course.

Solutions for Challenge: 1. ? = 11.5 2. ? = 6.2